## TEACHERS MAKING MEANING IN MATHS OR: WHAT DOES IT MEAN TO LEARN MATHS?

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The 15 year old boy next door is "no good" at maths. He failed. He got 6% in the exam after sitting there, for an hour a day, five days a week, 40 weeks in the year, for 4 years. He failed maths. Or, maths failed him. We know about those failures. We even expect them. But I also failed maths. After many years of apparent success and increasing abstraction, I admitted defeat. I didn't know how to ask questions, and how can you do a PhD without questions? Or did maths fail me too? What kind of discipline is it that produces, amongst its best, incurious students?

So, like many other teachers of mathematics, I am concerned about the large numbers of people who fail/do not achieve at / do not like mathematics. (Behind this question is another important question: does it matter? if so, how, and to whom?) As a teacher of teachers of maths, I have become more and more interested in what baggage intending teachers bring with them: what are their assumptions about maths, maths learning and maths teaching? In teaching intending teachers over the last four years, in two different institutions, I made an examination of these assumptions a focal point of the courses, by getting the students to keep a journal and to carry out a case-study. It is the case study that I shall discuss here, although the journal also provided valuable complementary insights.

For the case-study I have drawn on work done by Deborah Ball (1988) in the Department of Teacher Education at Michigan State University. She argues that if, as a constructivist perspective claims, learning is a product of the interaction between what is taught and what is brought to the learning situation, then this view could enrich our understanding not only of children's learning but of the learning of teachers themselves. She describes a project that she developed - for an introductory elementary teacher education course which was "designed to surface and challenge what entering elementary teacher candidates know about mathematics and how it is taught and learned." I see the case-study working on two levels: as a small but powerful intervention for the intending teachers, and as a source of valuable insights for those of us involved in teaching teachers. The more general insights - about maths, about learning - are not particularly new. It is more the consistency of misconceptions which underlie them and the struggles to change that intrigue me.

#### THE CASE-STUDY

Ball chose permutations as the subject matter of her project, on the basis that many of her students would not have studied it formally, and of those who had, most would be likely to have only a procedural understanding: they would know how to use the formulae. Unlike Ball's project, this case-study used subject-matter that all students had at some time studied formally: a problem about area and perimeter. The material used evolved out of work done by Vicky Webber and myself, with a variety of adult mathematics classes over a number of years. We have found the solving of this particular problem to be counter-intuitive for most students and particularly powerful in provoking heated arguments and bringing to the surface assumptions about mathematics and how it is learnt. By emphasising reflection and distancing the need for (at least mathematical) success, the case study allowed the

students to re-examine both the actual mathematics and the way we learn and teach it. Following Ball's basic framework, but with some variations, the case-study was constructed in three phases: learning maths; watching learning and teaching; and teaching. For the report the students, like the students in Ball's study, were not given any references about the mathematical content of the exercise, and were encouraged instead to read, if anything, more broadly in the area of learning. In particular they were given four references to look at: Eleanor Duckworth's *Teaching as research* (1987), David Hawkins' *I*, *Thou and It* (1984), John Holt's *How children fail* (1964), and Vivian Paley's *Wally's Stories* (1981). The students' notes said:

#### CASE STUDY OF LEARNING AND TEACHING MATHS

In tutorials, you will participate in a case study in which you yourself will first **learn** about aspects of area. A relevant video will be shown to give you a chance of **observing** children being taught related concepts. We then ask you to go home and **teach** these concepts to someone else - child or adultand finally to write a report on the whole case study.

In the report we would like you to include your experiences both as a learner and a teacher, and draw some tentative conclusions about mathematics, about the teaching and learning of mathematics and about learning to teach mathematics. The purpose of the study is **not** for you to be either a model learner or a model teacher at this point, but to reflect on how you and others learn maths, how you feel about it and why, and what implications this has for teaching maths.

phase one - learning: The actual mathematical focus of the study was:

A question: if you have a fixed length of fencing with which to make a holding pen for some sheep, will the sheep have the same total amount of grass inside the pen, no matter what shape you make it?

The students were given a moment or two, by themselves, to think about this question and to write down a quick spontaneous answer. This was followed by a series of related statements (eg if I increase the grass area of the pen, then I'll need more fencing as well; two pens with the same grassy area will need the same amount of fencing.....) which they were asked to read, noting down as they went an immediate true or false response to each statement. They were then asked to organise themselves into groups of three or four to discuss the statements and to try to come to a group consensus about the answers. If some members of the group did not understand, it was the responsibility of the others to convince them, using whatever means they found mutually acceptable. At least half an hour was spent in these small discussion groups, before the tutor asked the groups to share any findings or questions with the larger group. The role of the tutor was generally to extend or refocus questions rather than to answer them.

**phase two - watching learning and teaching:** The students were shown an Open University (UK) video, from the series Thinking Mathematically, in which primary children first come to grips with concepts of area at school. An experienced teacher is shown with several different age-groups of children, involved in activities from covering irregular pieces of paper with potato prints to finding a shortcut -a formula- for finding the area of rectangles.

**phase three - teaching:** Students tried to teach a person of their choice, from a wide range of ages, achievements and interests, what they themselves had learnt about area and perimeter.

#### LOOKING AT THE REACTIONS

#### The initial response

I was quite shocked to find out I was wrong when we began to experiment...When I saw the proof in the numbers I could see I was definitely wrong, but I couldn't see why....

The first reaction was an immediate "yes". After a few seconds of consideration doubt crept in....Subsequently a high degree of doubt formed.

In the first group, out of 200 people, about 80% - students and their students, old or young, men or women, studying university maths or not - got the answer wrong intuitively, ie almost everybody said, yes, if you've got a fixed amount of fencing, then you'll have a fixed amount of grass inside. Most of the few who said no, couldn't explain why, and only a few of the ones who could explain were able to do so without using a formula. Fewer than ten could really *see* why not. (Amongst those few were the three 5 and 6 year olds that students taught - are we teaching incompetence? creating blindness?) After discussion, the writing of the report gave the students a chance to examine their own reactions in some depth, and to reflect on their own underlying assumptions about maths and how it is learnt.

Similar results were obtained with students in other groups, in following years, from a different institution. The first group were the 108 students taking the course *Learning and teaching mathematics in schools* at Macquarie University in 1989, and included prospective secondary teachers, with a majority of intending primary teachers. The other three groups -approximately 50 people each year- were students enrolled in the Graduate Diploma in Adult Basic Education at the University of Technology, many of whom were already skilled teachers, predominantly in the literacy rather than numeracy area.

Why was 'intuition' so overwhelmingly inadequate, or even wrong? What can we learn from the intriguing consistency of the responses? What did the students learn from the whole experience?

## **IDEAS ABOUT AREA**

Trying to come to grips with why their "intuitive" responses had been so sure and so wrong, three students teased out their ideas of area:

[One of my assumptions] was that area is length x breadth, or whichever formula was appropriate for the shape.....The concept of area has been impressed on me in terms of the equations that give it value. My understanding of area wasn't developed from within me, I didn't own it. Rather I had adopted the formulas by rote. Area was a formula which was worked out by multiplying the length by the width. Shape appeared to have little to do with it. I believed if you adjusted the perimeter you must adjust the area. This seemed perfectly logical to me.

The thought never occurred to me that 'area' meant space inside a regionwhen we were finding area I never thought of it in terms of finding how much space was inside a certain region

for example:

finding that! ----> .

Instead I perceived that the concept of area involved a certain formula which was applied to a certain figure and ascertained that shape's 'area'...I remember thinking when watching the film -"why are they using potato shapes to display area to children?" It was not until our next tutorial that the point was made in discussion that the potato shapes were used to show children that the concept of 'area' involved space inside a certain region!

All three students point to the divorce they experienced between area as a concept with meaning, and area as a formula to be applied. For one of them, area *is* the formula; the fourth student was surprised when she found that the concept in fact could have meaning. Her experience of the divorce is reflected in the language she used: her first sentence, pointing to the realisation of meaning, flows naturally; the second, talking about rules, is awkward and formal. These students were by no means atypical, and were not confined to people who had dropped maths early, as this comment from a student studying university maths shows:

At school we had done a lot of maxima and minima problems involving area and perimeter, and if you were building rectangles the maximum area always turned out to be a square. I knew that the same perimeter did not always give the same area but I had never thought about why. I had never really worried about why the maximum area turned out to be a square - I had just thought it was a coincidence. With this activity however, I had to think about what area and perimeter actually were; my brain began to work overtime on what I had previously considered to be very simple. .....I had forgotten, if I had ever realized at all, that area was the amount of space that a shape covered and that perimeter was the distance around the shape.

Confronted time and time again with comments like these, I am astonished, not that so many people fail maths, but that so many pass it. With an understanding of area so fragile, so purely dependent on rule, so unanchored in the everyday world, the achievement of passing exams, of remembering which rule to apply to which set of words, is truly impressive. One of the very few students who gave a confident " no" answer to the main question firmly located his understanding within a practical context:

At about the same time my father asked me to design a new chicken pen. On presenting my elaborate rectangular design, he suggested a more square-shaped design, explaining that the area would be no different, but that the materials and man-hours would be less as the perimeter would be smaller.....Coming from the country I always conjured in my head a picture of a grassed area surrounded by a fence... Thus I had no trouble in reaching the conclusion that the longer and thinner a paddock was, the less area it enclosed....

## SOME MISLEADING ASSUMPTIONS

If we can work out why the responses of most of the intending teachers were so very uniform, and so very wrong, then we can begin to work what it is about their experience of maths and maths learning and teaching that has failed them. One common interpretation of the situation was to see the sheep pens only as rectangles; almost invariably this was done unconsciously rather than as an organising strategy:

...I had made an interesting assumption: I thought about the problem in terms of rectangles. This was also characteristic of the person I taught, and so raises some interesting questions. In my experience, most questions dealing with fenced areas involved rectangular or square fields...

Why did we as a group restrict ourselves to rectangles? Is it because the learned formulae  $A=L \times B$  and P = 2(L + B) are so deeply entrenched that it is hard to think of area in any other way? ......

I am [now] confident you can always find a larger perimeter enclosing a smaller area if you are not bound to maintain the shape of the figure eg



Another appeal was to something the students saw as "logic", which seemed to be based on an assumption of a direct, linear relationship between the two measures, perhaps deriving from their common dependence on 'L' and 'B', and the fact that they are usually taught together:

I had previously believed that area and perimeter were in some [direct] way interrelated, after all we had learnt about them simultaneously and I can never remember having been told that they were not related...

One of the first things I learnt... is the importance of not only teaching concepts but of relating them to each other... For instance R and I both made assumptions of the relationship between area and perimeter which show that we both knew about the two separate concepts but never really needed to relate the two (even after doing the HSC!)

I found that it was easy to get the wrong impression at first glance. It seemed logical that if the perimeter of the pen was increased then the area would increase as well and if we just think of rectangles then this is true.

Rectangles here are assumed to be all (mathematically) similar; this perception of rectangles being " the same shape" was common. Something like this diagram seems to be how the relationship is visualised, but never illustrated:



So, area *was* the formula; it was, somehow, to do with rectangles; perimeter and area were learnt side by side, but not together; and logic had little to do with meaning in the everyday world.

## IDEAS ABOUT MATHS MORE GENERALLY

The perceived chasm between the formal language of maths and more everyday meanings was not peculiar to the concept of area but was how students saw "doing maths" more generally:

When I first heard that I was required to learn some aspects of area, I felt reasonably confident and relaxed. From my experience, area problems were always solved by applying the relevant formula. As I remembered such formulae, I felt prepared to receive what I expected to be a work-sheet covered in shapes and measurements. When I received the actual sheet containing not a single number or shape, my feelings changed to uncertainty and confusion. Whilst clearly mathematical in nature, the questions asked could not be solved by using one of my answer-producing "recipes".... On reflecting on my reaction I have become aware of the underlying assumption I held, that maths was only numbers, rules and formulae.

There were variations, but the theme was inescapable:

The strangest thing happened when I first told S that we would be learning about area. Straight away he started muttering under his breath all the relevant formulas he could remember: "area of a rectangle equals length times breadth, area of a triangle equals half base times perpendicular height" etc. He finished by saying a confident "right" as he had proved to himself that he knew it ...more importantly, this showed quite clearly that mathematics to S was merely a collection of formulas and rules.

People are used to following set rules in maths, to the point where when they are given freedom to experiment they become suspicious and insecure. ..The formula had been taken away and considering I have come to rely solely on it to explain area, I felt marooned without it.

The image of maths as "a subject that is based on correctness", as another student put it - also came through strongly, though again not unexpectedly, in other reports:

For a subject that I had always felt contained only RIGHTS/WRONGS and reliable formulae, mathematics was proving very ...um...interesting.... challenging.

It had obviously not been part of M's maths experience to feel safe with guessing an answer before it was worked out. It seemed high on his priorities not to get an answer wrong.

I was too anxious to find the correct answer, to the point of being totally unconvinced about any of my own attempts.

One student described her attitude to maths as a belief in its "givenness", its formulas, its procedures, its expected answers, - woven together, for her, with some pleasure:

Maths has been a belief in a set of formulae. When you apply the formulae correctly to the question given you arrive at the "right" answer. This may sound completely unappealing, but in fact I enjoyed the game, its twists and turns and arriving at the correct answer. Maths was a static process. It was all written down in the book. You learnt it, you did it. I don't remember ever applying it to any part of my life....It also required another belief - belief in the authority of the teacher to be right.

Nowhere did students express any feeling that maths, like any other branch of knowledge, had been constructed by humans in response to human concerns and questions, that it had been something they themselves could make up and contribute to. For most, bridging the chasm between maths and their everyday lives was inconceivable.

## PERCEPTIONS AND MEMORIES OF LEARNING MATHS

Ideas about maths itself and ideas about how it was learnt, were tightly woven together: maths was formulas, learning maths was learning formulas, in an environment separate from "life":

The way it always was - teacher out the front, demonstrating some process, working a couple of examples on the board then having children work through examples in their books ...

... maths was learnt in set steps with set outcomes and creative thinking wasn't a concern..

No wonder I couldn't remember any maths. It had always consisted of learning formulae, using the correct process and coming up with "right" answers. It had never been relevant to my life. I'd never applied maths to everyday experiences. Not that the "mind games" of doing maths hadn't been enjoyable. They had. I had a good memory and I did well. But if I missed the sequence of the maths formulae, or, as in deductive geometry, you needed to rely on creative problem solving, I was lost.

Talking, discussion, was not part of doing this maths, either for the teacher....

"Get it clear in your mind, and have proof to show others, before you open your mouth," a teacher of mine used to say.

#### or for the pupils:

M. felt that often he knew how to do the maths but not how to use it, that a problem could be right before him that he knew the maths to solve if only the question wasn't disguised in words. If maths was taught the other way round, from the "I have a problem to solve" angle first, with the theory resulting from the investigation, maths might seem more relevant to students.

Disguise... the words seemed designed to catch you out....

My mind said, "Well, there has to be a catch - it'll be what I don't expect to happen, otherwise we wouldn't be doing it.... so ...."

I carried out this case study with my mother....she told me the first two statements were true, but changed her mind after drawing some diagrams. Then she said something which amazed me. ...She said, "Oh well, the rest of the questions seem true, but they must be false as well."

Learning and "doing" maths meant learning and applying formulas, working alone, looking out for tricks, with no expectation of meaning or connections with the everyday world.

# EFFECTS OF THE CASE-STUDY ON THESE VIEWS... OF MATHS, AND LEARNING

What was particularly impressive, and more hopeful, was the eagerness with which students worked, once they started to find meaning in what they were doing. Almost all the participants in the case-study became seriously engaged both with the mathematical question, and, through their attempted explanations of that question, with questions and insights about mathematics and their own learning. Some students had difficulty in owning a self in relation to mathematics, and in asking their own questions. This was reflected in an initial reluctance of some to use the word "I" in their reports and in their realization that they had hardly ever talked about their mathematical ideas. As the validity of their pictures of mathematics and learning were challenged by the experiences of the case-study, students recorded changing feelings and an understanding - even excitement - about the potential for change. Working in small groups, with discussion and concrete materials, raised the possibility of different, more interactive ways of learning, in which the learners could play some part in constructing mathematical meaning:

Having studied 3-unit mathematics at school and now doing maths at University, I expected to find the questions in this case study easy. Instead, I found them challenging. This is because investigating properties of area is something I had never been asked to do before.

I made an amazing discovery about maths during this case study ...I was finding that the subject matter was actually becoming relevant to me. I saw that maths could be effectively transformed to the level of the ordinary person in the classroom and was no longer a sacred subject which only those with a degree in this area could discover or alter.

One aspect of the relevance was a growing perception of the way in which maths grows out of, and feeds into, the everyday world:

[My student] to all intents and purpose would be considered a failure at maths. He was unsuccessful at maths in the HSC... [but] he had no problem with these questions. He answered them both quickly and correctly. The question that had caused me so much confusion was not at all confusing to him because he has spent time on farms and was able to give me an example as to where it would actually occur.... and I was supposed to be the "clever" one at maths! Some reports differentiated between merely doing maths, and understanding it:

If I am serious about becoming a maths teacher I am starting to realize that I am going to need to make sure that I understand what I am doing rather than simply doing it, which I feel I have been doing for years. I would like to break the cycle of students feeling that mathematics is just something you do -not something you understand. However first I am going to have to make sure that it is something that I understand.

I could see with my own eyes that indeed the area did change with equal perimeters but differently shaped areas. The experience for me was like the proverbial light bulb flashing on, but also one of frustration and anger. Why hadn't I been shown WHY before?

Students became aware that they became engaged in the mathematical question not so much when they made their initial response but when they tried to make sense of that response, either for themselves, or for others, particularly when responses in the group differed:

This may sound very strange but I think it was very fortunate that the answers obtained by members of our group were not the same. This meant we had to work through problems, over and over until we were all satisfied with the derived conclusion. It was not enough just to state your conclusion was right: I found I had to listen, query, agree and most importantly I had to attempt to explain my own understanding - not an easy task.

As we formed groups and discussed our intuitive answers to the questions, our cluster of women realized with some consternation that what had seemed previously reliable in our heads, clashed with the understanding of others and would therefore require 'real' working out, or 'proving'.

A central key in the development of meaning was, many students discovered, interaction, either with materials, or with other people. Talk began to be seen, not as cheating, but as a powerful way of constructing meaning:

My main AMAZING discovery in this tutorial clearly demonstrated the worth of ...the recommendation that students verbalise their thoughts and strategies. As I endeavoured to explain to my group (using fairly inept hand movements) how I imagined the fencing to be, using my own prior experiences with a guinea-pig holding, suddenly like a light-bulb, I was 'turned on' -I UNDERSTOOD!

As a student majoring in mathematics I feel that I am learning maths every day in my lectures, where I am told that some mathematician found a theorem, this is what it is, and this is how you use it. When however I sat down in my [case study] tutorial where we discussed our ideas and used examples to verify our results, you can imagine my confusion. I felt that I was and had been deceived in all my school and university maths education. I began to cast my mind back over all my other schooling experiences such as science classes where we did experiments and English where we discussed the themes of novels. My entire schooling appeared to occur in a sharing, interactive way except for maths. In these classes learning seemed to be a passive activity where you had to just absorb what you had been told. I was amazed when I found out in tutorial that this didn't have to be so. Learning can be defined as "the modification of behaviour through interaction with the environment" (Macquarie Dictionary). Mathematics, I realized from the case study, doesn't have to be the exception to this definition.

#### **CONCLUSION**

By presenting the students with a qualitative, relational question, involving them in working in small, relatively unthreatening groups, to convince each other of the reasonableness of their responses, and then requiring them to reflect on their reactions throughout, the case-study seems to have given the students an opportunity to re-view their ideas about maths and learning maths. The strong overall direction of change could be seen as a clear shift from the first to the second of Frankenstein's (1989) three ideologies of learning mathematics (adapted from Giroux's categorisation of ideologies underlying various approaches to native-language literacy [1982]). The first, instrumental ideology, sees knowledge as external, objective and neutral with instruction focusing on mastery of atomised content. The second, interactional ideology, sees knowledge as a human construction, with teaching focusing on meaning, being student paced, and emphasising problem-solving, process, and cognitive dissonance.

The separated formula-learning of the instrumental paradigm, so consistently described by the students, is sometimes referred to as abstraction, but we need to distinguish between the free-floating products of abstraction, and the process of abstracting. We abstract *from* some material reality, and in the impoverished vision of maths that we as a generation of teachers have given these students, there is no material reality to begin with: students simply expect, and are expected, to work with the already abstracted products. The more difficult, and powerful, achievement is that of actually abstracting. In the shift to interactional learning, this process element is beginning to emerge.

And surely the real power of abstraction itself lies in the possibility of its reconnection to the material world. It seems that the more abstract the subject the more status it has in our education system. Latin, physics, maths - study these for the HSC, and your marks soar. The separation of logic from meaning, however, encourages alienation from the everyday world and a terrible irresponsibility. Do the many mathematicians working in jobs funded by the military make any connection? The intending teachers in my study, a good proportion just out of high school, still saw maths in this passive, dangerous way. Could we not see the weaving of the abstract with the concrete as an even higher thinking? As Hegel (1975) says:

[the analytical approach] labours under a delusion if it supposes that while analysing the objects, it leaves them as they were: it really transforms the concrete into an abstract. And as a consequence of this change the living thing is killed: life can only exist in the concrete and one. Not that we can do without this division, if it is our intention to comprehend....The error lies in forgetting that this is only one half of the process, and that the main point is the reunion of what has been parted.

In an article comparing the maths that has evolved in a hunter-gatherer society with that of our western, industrial society Denny (1986) argues, that "the isolation and control of key

variables is needed for the high degree of alteration of the environment achieved by industrial society"; and, according to the mathematician Hyman Levy, "Pure mathematics is the method of isolation raised to a fine art." Perhaps what we mathematics educators should be valuing, however, is not only this skill of isolation, but also the skill involved in relating that abstract knowledge back to concrete experience, the skill of extracting from and putting back into context. Denny argues that, for the hunter, "inclusive knowledge of the whole pattern of natural processes will be imperative." We too, in our industrialised societies, with their proliferating global problems, increasingly complex and inter-related, may find that, as well as the fine art of isolation, the inclusive knowledge of inter-relating patterns will be essential to survival.

Thus, for me, the challenge is to work towards Frankenstein's third, critical ideology of mathematics learning, which includes a recognition of political conflict and power differences. It argues that learners actively generate their own knowledge from actual situations and a minimum of givens, and emphasises the ways in which maths can empower students to understand and act upon the world. This case-study has hardly begun to address the challenge of this critical framework for learning. Though quite powerful, the study is a very small intervention. However, without such beginnings, it is not likely that our future teachers will be any more concerned with connecting the abstract to the material world, with the accessibility of mathematics, with its potential for engendering critical awareness - with making meaning in mathematics - than our past ones. I would like to give the last word in this paper to one of those future teachers:

Immediately the question was placed in front of me my heart started to beat faster, my eyes raced across the words, and my mind immediately went searching for formulas, rules and mathematical jargon .. I came up against a brick wall, I could not think, nothing was registering, no immediate answer came to mind. ...I re-read the question...I tried to visualise...My thoughts were slowly moving from viewing area as length x breadth to thinking of area in terms of space inside a shape...Alas! alas! I find that I am what John Holt calls an answer-centred child "who sees a problem as a kind of announcement that far off in mysterious Answerland there is an answer that they are supposed to go and find." .....I found that discussion, taking time, using concrete materials and getting familiar with the problem rather than rushing into it made the learning process much easier and less frustrating....you need time to think and experiment. .... Although I truly love maths personally, I was scared by the thought that I would have to teach it. I was frightened that a pupil would say, "But Miss, I don't understand why", and I would have to say something like, "it just is - that's the rule", because I didn't fully understand the concept myself - I just knew the operations to follow and the rules to follow. I now realise that maths can be understood.....and I see maths as an integral part of everyday life not just some executive subject belonging to the geniuses of society.

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